FOSDEM 2015

Software Defined Radio devroom

Arithmetic based implementation of a quadrature FM Demodulator

SDR in GnuRadio

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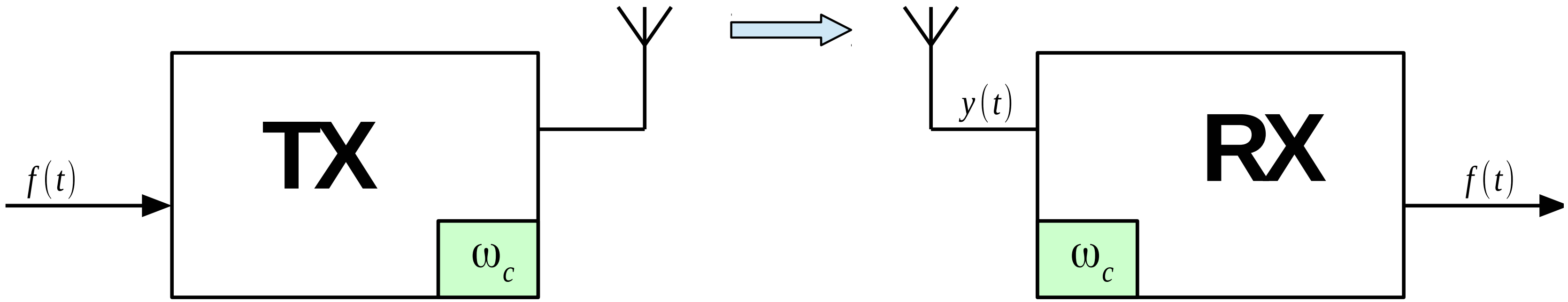
Overwiew:

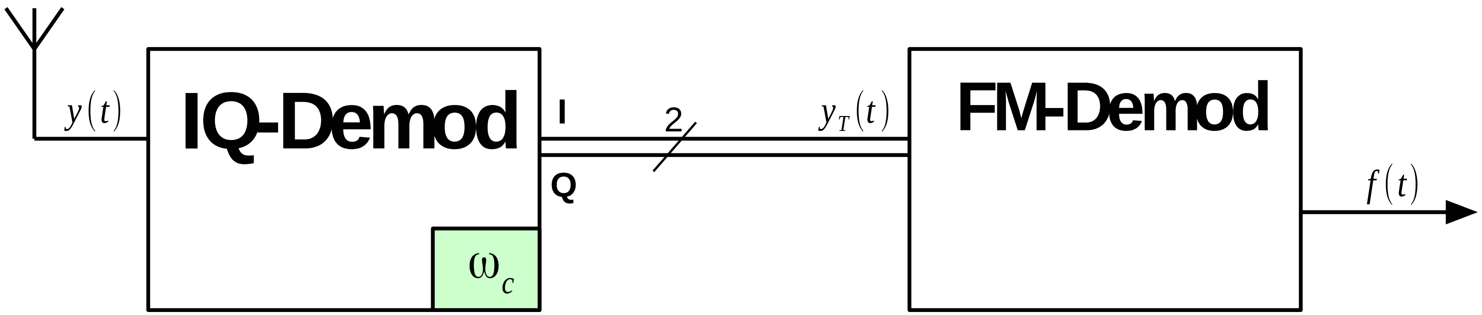
1. Background of angle modulation
2. Traditional FM-Demodulator
3. Arithmetical FM-Demodulator
4. Presentation of results
5. Bug fixing in *fast_atan*

Generall angle modulation in real world (RF)



ω_c	frequency of carrier, on which the modulated signal is transmited in the real world
$f(t) \in \mathbb{R}$	low frequency modulating signal
$y(t) \in \mathbb{R}$	real signal transmited by ω_c

Receiving of angle modulation



$y_T(t) \in \mathbb{C}$ transmited signal in low pass area

$$y_T(t) = \underbrace{y_{TR}(t)}_{I} + j \cdot \underbrace{y_{TI}(t)}_{Q} \quad (1)$$

Formal definition of Angle modulation

$$y(t) = A \cdot \cos\Big(\Phi(t)\Big) \tag{2}$$

PM : $$\Phi(t) = \varphi_0 + \alpha \cdot f(t) \tag{3}$$

FM : $$\Omega(t) = \frac{d\Phi(t)}{dt} = \dot{\Phi} = \omega_0 + \alpha \cdot f(t) \tag{4}$$

$$y(t) = A \cdot \cos\left(\omega_0 t + \alpha \cdot \int_{-\infty}^{t} f(\tau)\, d\tau + \varphi_0\right) \tag{5}$$

$$y_T(t) = A \cdot e^{j\left(\Delta\omega t + \alpha \cdot \int_{-\infty}^{t} f(\tau)\, d\tau + \varphi_0\right)} \tag{6}$$

Mathematical Background for a traditional FM-Demodulator (I)

Basic idea is the multiplication of current sample with conjugate complex version of preview sample.

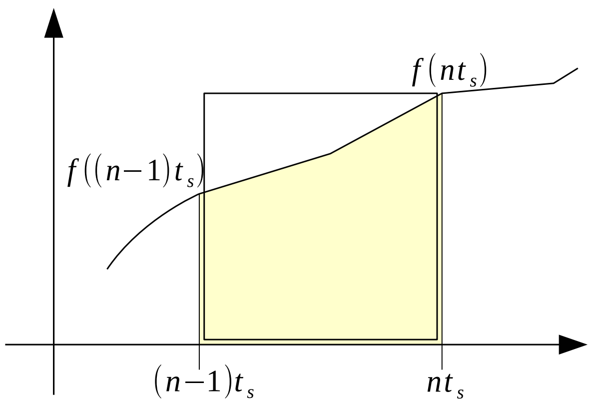
$$y_T(nt_s) \cdot y_T^*((n-1)t_s) \tag{7}$$

$$= A \cdot e^{j\left(\Delta\omega t_s n + \alpha \cdot \int_{-\infty}^{nt_s} f(\tau)\, d\tau + \varphi_0\right)} \cdot A \cdot e^{-j\left(\Delta\omega t_s (n-1) + \alpha \cdot \int_{-\infty}^{(n-1)t_s} f(\tau)\, d\tau + \varphi_0\right)}$$

$$= A^2 \cdot e^{j\left(\Delta\omega t_s + \alpha \cdot \int_{(n-1)t_s}^{nt_s} f(t)\, dt\right)} \tag{8}$$

Mathematical Background for a traditional FM-Demodulator (II)

The specific integral can be approximated by small value of t_s in following way:



$$\int_{(n-1)t_s}^{nt_s} f(t)\,dt \;\approx\; t_s f(nt_s)$$

Now we can simplify (8)

$$(8) = A^2 \cdot e^{j\left(\Delta\omega t_s + \alpha \cdot t_s \cdot f(nt_s)\right)} = A^2 \cdot e^{j\left(2\pi\Delta f t_s + 2\pi\text{dev} \cdot t_s \cdot f(nt_s)\right)}$$

Quadrature FM-Demod in GnuRadio

$$\text{out}(n) = \underbrace{\text{Gain}}_{\frac{1}{2\pi t_s}} \cdot \mathbf{arc}\left\{(y_T(n) \cdot y_T^*(n-1)\right\} = \Delta f + \text{dev} \cdot f(nt_s) \qquad (9)$$

"arc" implementation can be based on arctan, arccos, arcsin and needs often high computation effort.

Mathematical Background for an arithmetical FM-Demodulator (I)

Basic idea is the multiplication of derivation signal with with conjugate complex signal.

$$\dot{y}_T(t) \cdot y_T^*(t) \tag{10}$$

$$\dot{y}_T(t) = j \cdot A \cdot \left(\Delta\omega + \alpha \cdot f(t)\right) \cdot e^{j\left(\Delta\omega t + \alpha \cdot \int\limits_{-\infty}^{t} f(\tau)\, d\tau + \varphi_0\right)}$$

$$y_T^*(t) = A \cdot e^{-j\left(\Delta\omega t + \alpha \cdot \int\limits_{-\infty}^{t} f(\tau)\, d\tau + \varphi_0\right)}$$

$$\Rightarrow \quad \dot{y}_T(t) \cdot y_T^*(t) = j \cdot A^2 \left(\Delta\omega + \alpha \cdot f(t)\right) \tag{11}$$

Mathematical Background for an arithmetical FM-Demodulator (II)

$$\dot{y}_T(t) \cdot y_T^*(t) = (i' + j \cdot q') \cdot (i - j \cdot q)$$

$$= \underbrace{(i' \cdot i + q' \cdot q)}_{=0} + j \cdot (i \cdot q' - i' \cdot q) \stackrel{!}{=} j \cdot A^2 \Big(\Delta\omega + \alpha \cdot f(t) \Big)$$

$$\Rightarrow \quad (i \cdot q' - i' \cdot q) = A^2 \Big(\Delta\omega + \alpha \cdot f(t) \Big) \qquad (12)$$

Mathematical Background for an arithmetical FM-Demodulator (III)

Now let us assume that

$i_n := i(nt_s) \qquad i_{n-1} := i((n-1)t_s)$

$q_n := q(nt_s) \qquad q_{n-1} := q((n-1)t_s)$

Now we can write the left site of (12) as follow

$$\begin{aligned} i \cdot q' - i' \cdot q &= i_n \frac{q_n - q_{n-1}}{t_s} - \frac{i_n - i_{n-1}}{t_s} q_n \\ &= \frac{1}{t_s}(i_n q_n - i_n q_{n-1} - i_n q_n + i_{n-1} q_n) \\ &= \frac{1}{t_s}(i_{n-1} q_n - i_n q_{n-1}) \qquad (13) \end{aligned}$$

Mathematical Background for an arithmetical FM-Demodulator (IV)

Now we put the (13) into (12) and obtain:

$$\frac{1}{t_s}(i_{n-1}q_n - i_n q_{n-1}) = A^2\Big(\Delta\omega + \alpha \cdot f(nt_s)\Big)$$

$$\frac{1}{2\pi t_s \cdot \underbrace{(i_n^2 + q_n^2)}_{A^2}}(i_{n-1}q_n - i_n q_{n-1}) = \Delta f + \text{dev} \cdot f(nt_s) \tag{14}$$

Now we can rewrite this result similar like (9)

$$\text{out}(n) = \underbrace{\text{Gain}}_{\frac{1}{2\pi t_s}} \cdot \frac{i_{n-1}q_n - i_n q_{n-1}}{i_n^2 + q_n^2} = \Delta f + \text{dev} \cdot f(nt_s) \tag{15}$$

Now both Demodulators in cartesian form

"arc" based FM-Demodulator:

$$\begin{aligned}\text{out}(n) &= \underbrace{\text{Gain}}_{\frac{1}{2\pi t_s}} \cdot \mathbf{arc}\left\{(i_n i_{n-1} + q_n q_{n-1}) + j \cdot (q_n i_{n-1} - i_n q_{n-1})\right\} \\ &= \Delta f + \text{dev} \cdot f(nt_s) \qquad (16)\end{aligned}$$

"arithmetic" based FM-Demodulator:

$$\text{out}(n) = \underbrace{\text{Gain}}_{\frac{1}{2\pi t_s}} \cdot \frac{i_{n-1} q_n - i_n q_{n-1}}{i_n^2 + q_n^2} = \Delta f + \text{dev} \cdot f(nt_s) \qquad (17)$$

Comparison on computation performance

"arc" Implementaion	GENERIC	SSE3
	11,28s	9,90s

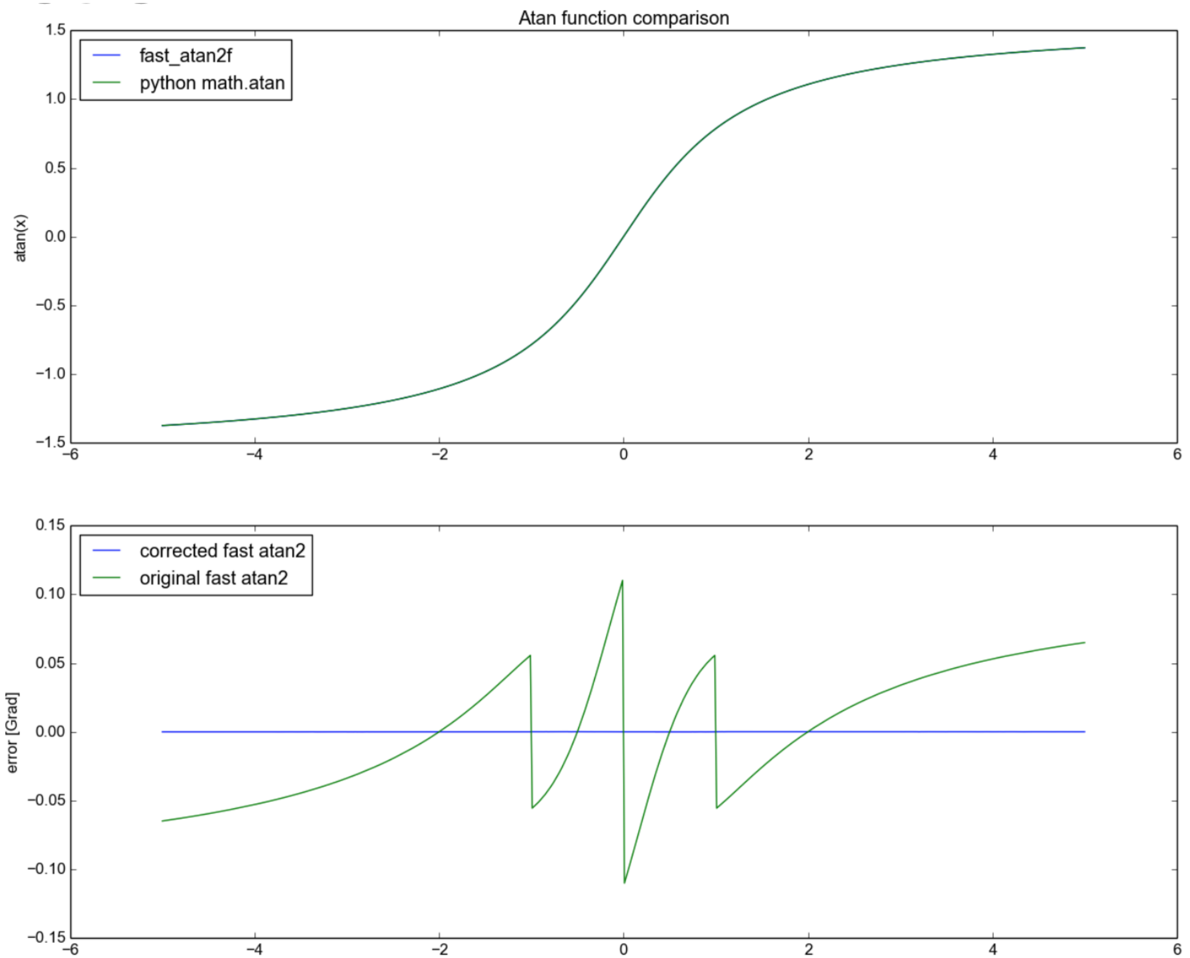
"arithmetic" Implementation	GENERIC	SSE3
1 thread	3,10s	1,80s
2 threads	1,65s	1,04s
3 threads	1,23s	0,81s
4 threads	0,98s	0,65s

Times are measured for computation
of some millions of samples on a 4 core i7-CPU.

Visualization of demodulation error

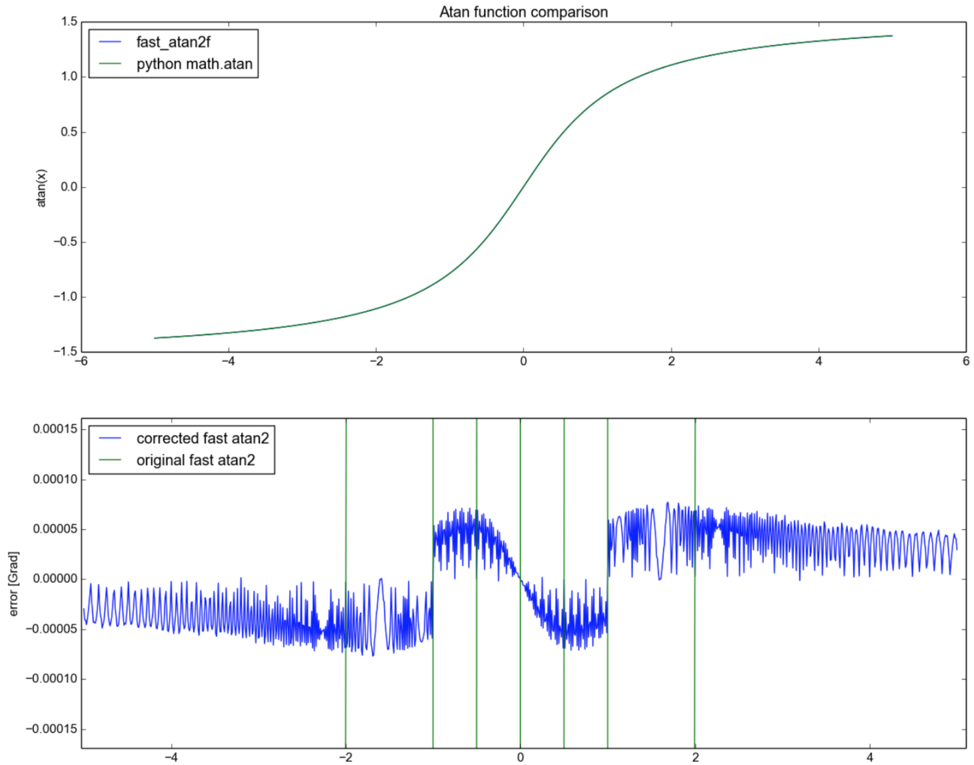
Deviation in Hz	f_{mod} in Hz	f_{s} in Hz	$\varepsilon_{\mathrm{arc}}$ in dB	$\varepsilon_{\mathrm{arith}}$ in dB
1000	10	8000	-107	-22
1000	10	14000	-111	-32
1000	10	25000	-111	-42
1000	10	45000	-111	-52
100	10	8000	-105	-62
100	100	8000	-72	-60
100	400	8000	-48	-46
1000	1000	8000	-32	-20
1200	2400	28800	-39	-34

Bug fixing in the fast_atan implementation I



$$\text{error}(x) := \text{fast\_atan}(x) - \arctan(x)$$

Bug fixing in the fast_atan implementation II



In the original Version the maximal error was below 0.111°, in the fixed version the error is below $8.2 \cdot 10^{-5\circ}$

Questions?

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